

# Chapter 2 Pythagorean Theorem II Applications and the Converse

## Teacher Guide and Diagnostic Answer Key

### Teacher Guide

#### Teacher Chapter-at-a-Glance

|               |
|---------------|
| None required |
| Pacing        |
| Talk Prompt   |

### Mastery Decision Guide

A missed Tripwire does not automatically mean “redo the chapter.” Confirm the likely misconception, reteach the smallest prerequisite or representation that addresses it, and then use a fresh item before moving on.

Missed Tripwire #1 → The hypotenuse was not identified before choosing the operation. Next move: Mark  $c=25$  first and use  $c^2-a^2=b^2$  for a missing leg.

#### Confirmation and Fresh Recheck

Confirm with: Ask the student to label  $c$  first and say “hypotenuse missing, leg missing, or converse?” before choosing the operation.

Fresh recheck: Give one missing-leg problem and one converse check using new numbers.

Decision rule: Move forward only when the fresh response is secure and the reasoning survives the changed representation or context.

### Diagram Production Notes — Chapter 2

Use the diagram master IDs:

Ch13-D01 — Missing Hypotenuse

Ch13-D02 — Missing Leg

Ch13-D03 — Add or Subtract Decision Box

Ch13-D04 — Ladder Against Wall

Ch13-D05 — Ramp Diagram

Ch13-D06 — Rectangle Diagonal

Ch13-D07 — Converse Check

Ch13-D08 — Coordinate Grid Distance

The missing-leg diagram must clearly show the hypotenuse already labelled. The decision box should be simple and bold enough for students to use as a reference. Application diagrams must show the right angle clearly. Coordinate-grid distance diagrams must show horizontal and vertical legs before the diagonal.

## Ready to Learn Check

Quick check before starting. A wrong response below points to a prerequisite worth confirming before the chapter builds on it.

1. In a right triangle, is the hypotenuse the longest side?

Correct answer: Yes.

Likely wrong answer: No.

Possible misconception: The hypotenuse property of a right triangle is not secure.

Next move: Mark the right angle and compare the side opposite it with the legs.

2. If the hypotenuse is missing, do you add or subtract the leg squares?

Correct answer: Add the leg squares.

Likely wrong answer: Subtract.

Possible misconception: The missing-side case has not been identified.

Next move: Write “hypotenuse missing → add” above the setup.

3. If a leg is missing, do you add the hypotenuse square or subtract from it?

Correct answer: Subtract a known leg square from the hypotenuse square.

Likely wrong answer: Add the hypotenuse and leg squares.

Possible misconception: The student is using the hypotenuse-missing routine for a missing leg.

Next move: Write  $c^2 - a^2 = b^2$  before substituting numbers.

Ready-to-Learn item 1

Correct answer: Yes.

Likely wrong answer: No.

Misconception: The hypotenuse property of a right triangle is not secure.

Next move: Mark the right angle and compare the side opposite it with the legs.

Ready-to-Learn item 2

Correct answer: Add the leg squares.

Likely wrong answer: Subtract.

Misconception: The missing-side case has not been identified.

Next move: Write “hypotenuse missing → add” above the setup.

Ready-to-Learn item 3

Correct answer: Subtract a known leg square from the hypotenuse square.

Likely wrong answer: Add the hypotenuse and leg squares.

Misconception: The student is using the hypotenuse-missing routine for a missing leg.

Next move: Write  $c^2 - a^2 = b^2$  before substituting numbers.

## Practice

*Assign the full set or select items for targeted review. Teacher solutions follow each question.*

**1. A right triangle has legs 7 cm and 24 cm. Find the longest side.**

**Solution:** 1. hypotenuse missing

**2. A right triangle has hypotenuse 13 m and one leg 5 m. Find the other leg.**

**Solution:** 2. leg missing

**3. A triangle has side lengths 8, 15, and 17. Decide whether it is right-angled.**

**Solution:** 3. checking

**4. A ladder leans against a wall. The wall height and ground distance are given. Find the ladder length.**

**Solution:** 4. hypotenuse missing

**5. A ramp length and height are given. Find the horizontal distance.**

**Solution:** 5. leg missing

**6. legs 7 cm, 24 cm; find hypotenuse**

**Solution:** 6.  $7^2 + 24^2 = c^2$   $49 + 576 = c^2$   $625 = c^2$   $c = 25$  The hypotenuse is 25 cm.

**7. legs 10 m, 24 m; find hypotenuse**

**Solution:** 7.  $10^2 + 24^2 = c^2$   $100 + 576 = c^2$   $676 = c^2$   $c = 26$  The hypotenuse is 26 m.

**8. hypotenuse 13 cm, leg 5 cm; find missing leg**

**Solution:** 8.  $5^2 + b^2 = 13^2$   $25 + b^2 = 169$   $b^2 = 144$   $b = 12$  The missing leg is 12 cm.

**9. hypotenuse 25 m, leg 7 m; find missing leg**

**Solution:** 9.  $7^2 + b^2 = 25^2$   $49 + b^2 = 625$   $b^2 = 576$   $b = 24$  The missing leg is 24 m.

**10. hypotenuse 17 km, leg 8 km; find missing leg Use the longest side as c.**

**Solution:** 10.  $8^2 + b^2 = 17^2$   $64 + b^2 = 289$   $b^2 = 225$   $b = 15$  The missing leg is 15 km.

**11. 8, 15, 17**

**Solution:** 11.  $8^2 + 15^2 = 64 + 225 = 289$   $17^2 = 289$  It is right-angled.

**12. 7, 24, 25**

**Solution:** 12.  $7^2 + 24^2 = 49 + 576 = 625$   $25^2 = 625$  It is right-angled.

**13. 9, 12, 14**

**Solution:** 13.  $9^2 + 12^2 = 81 + 144 = 225$   $14^2 = 196$  It is not right-angled.

**14. 10, 11, 15 Ch13-D07 — Converse Check**

**Solution:** 14.  $10^2 + 11^2 = 100 + 121 = 221$   $15^2 = 225$  It is not right-angled.

**15. A ladder is 13 m long. The bottom of the ladder is 5 m from the wall. How high up the wall does the ladder reach?**

**Solution:** 15. The ladder is the hypotenuse.  $5^2 + b^2 = 13^2$   $25 + b^2 = 169$   $b^2 = 144$   $b = 12$  The ladder reaches 12 m up the wall.

**16. A rectangular field is 30 m long and 40 m wide. A student walks diagonally across it. Find the diagonal distance.**

**Solution:** 16.  $30^2 + 40^2 = c^2$   $900 + 1600 = c^2$   $2500 = c^2$   $c = 50$  The diagonal distance is 50 m.

**17. A ramp rises 6 m and has a horizontal run of 8 m. Find the ramp length.**

**Solution:** 17.  $6^2 + 8^2 = c^2$   $36 + 64 = c^2$   $100 = c^2$   $c = 10$  The ramp length is 10 m.

**18. On a coordinate grid, point A is at (1,2) and point B is at (7,10). Find the distance from A to B.**

**Solution:** 18. Horizontal distance:  $7 - 1 = 6$  Vertical distance:  $10 - 2 = 8$  Distance:  $6^2 + 8^2 = c^2$   $36 + 64 = c^2$   $100 = c^2$   $c = 10$  The distance from A to B is 10 units.

**19. A student adds  $17^2 + 8^2$  when the hypotenuse is already 17. Explain the mistake.**

**Solution:** 19. The hypotenuse is already 17, so  $17^2$  should be on the right side of the equation. The student should subtract to find the missing leg.

**20. A student checks a triangle with side lengths 6, 8, and 9 using 6 as c. Explain the mistake.**

**Solution:** 20. When checking a triangle, the longest side must be c. The student should use 9 as c, not 6.

**21. A student says a rectangle diagonal is the same length as the rectangle's width. Explain why that is wrong.**

**Solution:** 21. The diagonal crosses the rectangle from corner to corner. It is the hypotenuse of a right triangle, so it is longer than the width alone.

## Chapter Answer Key

Tripwire Check

The student is not correct.

The hypotenuse is already given:

25 cm

Let the missing leg be b.

$$7^2 + b^2 = 25^2$$

$$49 + b^2 = 625$$

$$b^2 = 625 - 49$$

$$b^2 = 576$$

$$b = \sqrt{576}$$

$$b = 24$$

The missing leg is:

24 cm

## Worksheet 1: Guided Practice

1.

a. add

b. subtract

c. subtract

2.

$$c = 15$$

The hypotenuse is:

15 cm

3.

$$b^2 = 144$$

$$b = \sqrt{144}$$

$$b = 12$$

The missing leg is:

12 cm

4.

The longest side is 5.

$$3^2 + 4^2 = 25$$

$$5^2 = 25$$

Yes, it is right-angled.

## Worksheet 2: Independent Practice

1.

a.

$$6^2 + 8^2 = 100$$

$$c = 10$$

10 cm

b.

$$9^2 + 12^2 = 225$$

$$c = 15$$

15 m

c.

$$12^2 + 16^2 = 400$$

$$c = 20$$

20 km

d.

$$15^2 + 20^2 = 625$$

$$c = 25$$

25 cm

## 2.

a.

$$6^2 + b^2 = 10^2$$

$$36 + b^2 = 100$$

$$b^2 = 64$$

$$b = 8$$

8 cm

b.

$$9^2 + b^2 = 15^2$$

$$81 + b^2 = 225$$

$$b^2 = 144$$

$$b = 12$$

12 m

c.

$$15^2 + b^2 = 25^2$$

$$225 + b^2 = 625$$

$$b^2 = 400$$

$$b = 20$$

20 km

d.

$$10^2 + b^2 = 26^2$$

$$100 + b^2 = 676$$

$$b^2 = 576$$

$$b = 24$$

24 cm

## 3.

a. right-angled

b. right-angled

c.

$$7^2 + 9^2 = 49 + 81 = 130$$

$$12^2 = 144$$

not right-angled

d.

$$9^2 + 40^2 = 81 + 1600 = 1681$$

$$41^2 = 1681$$

right-angled

**4.**

a.

$$8^2 + 15^2 = 289$$

$$c = 17$$

Diagonal: 17 cm

b.

$$5^2 + 12^2 = 169$$

$$c = 13$$

Ramp length: 13 m

c.

$$b^2 + 8^2 = 10^2$$

$$b^2 + 64 = 100$$

$$b^2 = 36$$

$$b = 6$$

The bottom is 6 m from the wall.

### Worksheet 3: Mixed Practice & Application

**1.**

a. Hypotenuse missing.

$$20^2 + 21^2 = c^2$$

$$400 + 441 = c^2$$

$$841 = c^2$$

$$c = 29$$

The hypotenuse is 29 cm.

b. Leg missing.

$$20^2 + b^2 = 29^2$$

$$400 + b^2 = 841$$

$$b^2 = 441$$

$$b = 21$$

The missing leg is 21 m.

c. Checking.

$$10^2 + 24^2 = 100 + 576 = 676$$

$$26^2 = 676$$

The triangle is right-angled.

## 2.

a. The ladder, wall, and ground form a right triangle.

b. The ladder is the hypotenuse because it is across from the right angle.

c.

$$9^2 + b^2 = 15^2$$

$$81 + b^2 = 225$$

$$b^2 = 144$$

$$b = 12$$

The ladder reaches 12 m up the wall.

## 3.

$$16^2 + 30^2 = c^2$$

$$256 + 900 = c^2$$

$$1156 = c^2$$

$$c = 34$$

The diagonal is 34 cm.

## 4.

a.

$$8 - 2 = 6$$

Horizontal distance is 6.

b.

$$11 - 3 = 8$$

Vertical distance is 8.

c.

$$6^2 + 8^2 = c^2$$

$$36 + 64 = c^2$$

$$100 = c^2$$

$$c = 10$$

The distance from A to B is 10 units.

## 5.

The student changed the side length 14 into 15. The triangle must be checked using the actual side lengths:

$$9^2 + 12^2 = 225$$

$$14^2 = 196$$

Since  $225 \neq 196$ , the triangle is not right-angled.

**6.**

When checking a triangle, the longest side must be used as  $c$ . For 6, 8, and 10, the longest side is 10.

$$6^2 + 8^2 = 36 + 64 = 100$$

$$10^2 = 100$$

The triangle is right-angled.

## Diagnostic Key — High-Leverage Items

### Tripwire Check

**Correct answer:** No. Since 17 cm is the hypotenuse, subtract:  $b^2 = 17^2 - 8^2 = 225$ , so  $b = 15$  cm.

**Likely wrong answer:** Using  $17^2 + 8^2$ , or an answer larger than 17 cm.

**Misconception:** The hypotenuse was not identified before choosing the operation.

**Next move:** Mark  $c = 17$  first and use  $c^2 - a^2 = b^2$  for a missing leg.

### Tripwire Diagnostic

Correct answer: No. Since 25 cm is the hypotenuse, subtract:  $b^2 = 25^2 - 7^2 = 576$ , so  $b = 24$  cm.

Likely wrong answer: Using  $25^2 + 7^2$ , or an answer larger than 25 cm.

Misconception: The hypotenuse was not identified before choosing the operation.

Next move: Mark  $c = 25$  first and use  $c^2 - a^2 = b^2$  for a missing leg.

## Exit Ticket Diagnostic — High-Leverage Item

Correct answer: Yes;  $7^2 + 24^2 = 25^2$ .

Likely wrong answer: No, or testing a shorter side as  $c$ .

Misconception: The longest side was not identified as the possible hypotenuse before the converse check.

Next move: Circle 25 as  $c$ , then compare  $7^2 + 24^2$  with  $25^2$ .

## Variation Lab — Expected Ideas

1. Identify  $c$  first. Missing hypotenuse → add squares; missing leg → subtract from  $c^2$ .
2. A rectangle diagonal creates a right triangle with the side lengths as legs.
3. For the converse, the longest side must be tested as  $c$  and the square sums compared exactly.

## Transfer Check — Suggested Responses

Familiar:  $\sqrt{169 - 25} = 12$  m. Altered:  $\sqrt{324 + 576} = 30$  cm. Non-routine: 10 units. Repair:  $17^2 - 8^2 = 225$ , so the missing leg is 15 cm.

## Exit Ticket — Answers

1. 12 m

2. Yes:  $7^2 + 24^2 = 25^2$

3. Because  $c^2$  is the largest square; for a missing leg subtract the known leg square from  $c^2$ .